A Dynamical Approach for Modelling and Control of Production Systems

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Abstract. Production systems are considered to be dynamical systems. A dynamical approach for modelling and control of production systems evolves out of the context of new developments in production research. A model of an elementary production system is set up in order to demonstrate the idea of control in the introduced approach. Some simulation results are given.

INTRODUCTION

The development of Nonlinear Dynamics and its applications enables us to understand and describe complex systems where linear approaches fail or are too far from reality. Complex dynamic behaviour can occur in relatively simple production systems. Beaumariage and Kempf have shown the sensitive dependence of throughput times on the initial conditions and scheduling rules in a production system model [1]. The origin of this unstable behaviour is not obvious. Other authors find even contrary results [2]. Bartholdi, Bunimovich and Eisenstein have shown, that deterministic models can describe the dynamics of production lines appropriately [3][4]. Understanding the production dynamics is therefore a fundamental part in a dynamical approach for modelling and control of production systems.

Classical production planning and control systems (PPC systems) are based on concepts that do not consider the production system as a dynamical system. Usually, heuristic approaches are preferred in order to simulate the production process and its scheduling and control. But optimisation methods do not provide the controller with good results if there are some changes during the optimisation period [5][6]. In the context of today’s highly dynamic market with its rapid changes in demand, flexibility and variety are recognized as increasing measures of modern PPC systems.

In the third section of this paper, a dynamical point of view is introduced, which incorporates dynamical aspects of a concept level of PPC, taking into account functional aspects of PPC and modelling of production systems as well.
PRODUCTION SYSTEMS AND PPC CONCEPTS

Production systems can be classified as capacity oriented production vs. customer oriented production. Customer oriented production systems are considered to have a high degree of flexibility. They need information feedback from the shop floor in order to influence the product flow in short time intervals in a non-periodic, order dependent way to meet the needs of a highly dynamic market. This type of production system is considered in this paper.

The production planning and control in an enterprise consists of the following problems:

- Production program planning
- Material requirements planning
- Throughput time scheduling
- Capacity requirement planning
- Order release
- Order monitoring and control

All of these problems have to be solved by a PPC system, which is able to run the entire manufacturing process. How they are solved, in what order, if local or as part of a global strategy, depends on the PPC concept underlying the PPC system. In general, PPC systems are designed to find optimal solutions for specific PPC problems. The most important optimisation goals are throughput times, total costs, capacity utilization, inventory costs and delivery reliability. Thereby, strategies for different objectives can lead to contrary results, e.g. maximal capacity utilization and minimal throughput times.

Models of production systems are used to put these strategies into practice. They are derived from classical optimisation tasks studied in Operations Research [7]. The model for the production system itself therefore consists of different optimisation problems. There is no interaction of these particular theories. This could be developed in a meta-theory, but in general, the results will be hard to interpret [8].

Most recent PPC systems work in line with successive planning concepts consisting of successive planning steps mentioned above. It is widely used but also criticised because it lacks the interaction between the planning steps [8]. Several approaches try to overcome the difficulties resulting from this planning and control scheme [8][9]. They extend the possibilities of traditional PPC systems towards faster adaptation to changes in demand.
THE DYNAMICAL APPROACH

Why a Dynamical Approach?

Known concepts in PPC are normally founded on models for parts of PPC that solve local problems like the optimisation function of lot sizes [10]. Optimisation models (solved by exact algorithms or heuristic approaches), as discussed in Operations Research, are often not satisfying even at a low level of complexity, e.g. 3 machines, 2 job steps, more than 2 products [10]. They do not describe the qualitative behaviour.

Dynamical models are different from optimisation models as they describe relations between system variables. The main principle is understanding the dynamic behaviour instead of finding solutions for particular problems. To get predictable results, an intrinsic deterministic model is necessary, into which stochastic influences can be incorporated later on. Bartholdi, Bunimovich and Eisenstein have shown that sewing production lines can be described by way of a deterministic model [3][4]. The dynamic behaviour was driven exclusively by deterministic rules.

In a more complex production system, a number of deterministic processes occur. A classification of the functional aspects of PPC is therefore necessary. It will be discussed in the following sections.

The Idea of the Dynamical Approach

Rapid adaptation to frequent changes in demand and product mix is required in a customer oriented production system. The point of view put forward in this paper is the combination of ideas founded on the Nonlinear Dynamics Theory and PPC research into a new approach of modelling and control.

The dynamical approach is a generalized approach which includes modelling of manufacturing processes and a dynamical control mechanism. This includes analysing dynamic behaviour according to Nonlinear Dynamics and solving PPC problems as well. The starting point for the control of chaos is the phase space of the system, which is spread out by the system variables. Several methods for the control of chaos have been developed [11].

The dynamics of particular manufacturing processes is caused by rules and conditions, which are parts of the functional structure of the PPC system. The dynamical approach consists of 3 levels of description:

1. Modelling of the production system
2. Dynamical control
3. Functional structure
Figure 1 shows the interaction of level 1 (product flow) and level 2 (information flow). A system state is defined which represents the actual situation in the production system. It is derived from the state space of the system, which contains the time series of measured data in analogy to the phase space of physical systems. Applications to production systems use variables related to the work content in the system like the so-called “work-in-process” (WIP), a widely used key parameter of manufacturing.

Information feedback from the shop floor and the arrival of new orders update the system state. This can be put into practice by frequent measurements of system variables like buffer levels, WIP of machines or system specific variables. To determine the system state, further analysis of the data is necessary in complex realistic models. This can be done by means of Nonlinear Dynamics Theory. In simple cases, the immediate construction of state spaces from the time series can reveal the dynamics of the system. An example will be given in the last section of this paper.

The control mechanism consists of 3 steps:

1. The System is in equilibrium.
2. Disturbances occur.
3. A new equilibrium is reached after dynamic adjustment of the system.

Orders are regarded as disturbances, leading to a new system state which includes reorganisation of order sequences.

In Level 3, basic functional aspects are collected in independent functional groups that enable the system to work. They are at first a framework for modelling the production system and provide finally possibilities to control the production process by
a controller or by the system itself. The latter case is a step towards self-control, which is a fundamental idea in the dynamical approach.

The following functional groups are defined:

- The structure
- The capacity
- The operational rules
- The order release
- The queuing policies

These functional groups generate the dynamics of a production system and enable and influence the product flow through the system. They are discussed in detail in the next section.

**The Functional Groups in the Dynamical Approach**

*The Structure*

The functional group “structure” contains information about the number and arrangement of work stations, machines, buffers or storages necessary to describe the manufacturing process sufficiently. The hierarchy of structural information allows a more or less detailed description depending on the modelling aims.

The structure also contains information about the product flow. It shows how different types of products can be put through a system. This becomes relevant, for instance, in parallel manufacturing facilities where it is possible to do the same job at different machines. The structure of the product flow is determined by the layout of the shop floors and by the process plans of every product in the production program. In a typical job shop production system, one finds a complex situation with a number of machines arranged according to their functions, e.g. turning, milling, drilling etc. Therefore, the product flow contains convergences, division and feedback loops.

The structure of the product flow is considered as a function of the structure and state of the production system. Changes in the structure, e.g. caused by machine replacements, immediately influence the structure of the product flow and system state due to the dynamic interactions instead of new planning steps.

*The Capacity*

The capacity of a production system includes three important aspects: the capacity of space, the capacity of time and the capacity of manufacturing. The capacity of space describes the physical space to store and manufacture raw material, sub-assemblies and final products, e.g. storages and buffers. Capacity of time means the working time in a day, e.g. 8 or 16 hours per working day. The capacity of manufacturing contains the
volume of production, product variety, quality measures and other parameters depending on machine parameters and structure.

Capacity is usually considered as a parameter (or a set of parameters) in production systems characterized by upper and lower limits. In a number of models there are no capacity limitations. In the dynamical approach, a detailed description of capacity is necessary to enable the system to adjust itself in a flexible way. Therefore, all three capacities are considered as variables that can be continuously modified.

So the role of storages and buffers could move from a passive box with fixed upper and lower limits to an active element. The flexible handling of the capacity of time (overtime work, part-time work) is a common procedure to balance costs and demand. The capacity of manufacturing is a variable value if at least one of the processes affecting this capacity changes over time. An example for a large scale change is a growing product variety that causes additional setup processes at the machines. This increases the throughput times and so decreases the capacity to manufacture these products. Examples of small scale changes to the capacity of manufacturing are machine breakdowns or missing tools, late deliveries of parts or rush orders.

The Operational Rules

If demand exists and manufacturing orders are waiting to be released and processed, the operational rules generate the dynamics of a production system. Such rules are generally simple handling processes but need to be defined in detail. An example of an operational rule is: what happens in a machine if the input buffer is empty or if the output buffer is full and how does this effect the adjacent machines and buffers.

There are some investigations that identify and eliminate such constraints in a production system [12]. In the majority of cases, the order and material flow and the position of the constraint are considered to be constant and static. This assumption is appropriate for a flow shop production system with a constant demand and a constant product mix. But for a job shop, changing demand and customer-specific products are typical. In this case, the order and material flow and therefore the position of constraints vary over time.

The dynamical approach suggests a dynamic adjustment of the operational rules on the current situation in the production system. An example is given in the last section of this paper.

The Order Release

In conventional PPC systems, the production orders are released after the throughput time scheduling, the capacity planning and the order sequencing. Order sequence and release dates are more or less fixed. Such a production schedule assumes constant throughput times and capacity availabilities during the entire planning period.
If rush orders must be released or resources fail, the fixed production schedule becomes void.

This problem is well-known and has been investigated as the successive concept for PPC fails against the background of a highly dynamic market with a growing product variety, customer specific products and short delivery times [13][14].

The dynamical approach suggests a dynamic order release depending on both the incoming customer orders and the actual situation on the shop floor.

The Queuing Policies

Queuing policies are rules that determine the withdrawal of material from buffer to manufacture on a machine. A well-known and widely used queuing rule is “first in - first out”. Thereby, the processing sequence of the orders remains constant. Dynamic queuing rules like “least average static slack” or “least average dynamic slack” cause withdrawal depending on the stage of processing and due-date. But they do not take into account the situation at the work stations.

The dynamical approach allows the choice between different queuing rules. For the control of the order flow under quickly changing conditions, the dependence on the system state will play an important role.

A DYNAMICAL MODEL OF A PRODUCTION SYSTEM

The Boundary Conditions

The following sections will demonstrate the function of operational rules which influence the production process depending on the system state. The operational rules are considered dynamic objects. All other functional groups are kept constant. The goal is to control the production system via its system state by means of parameter changes in the operational rules.

Structure: An elementary production system with two work stations as shown in figure 2 will be considered. The structure of the product flow is linear with one input and one output channel assigned to each work station.

\[ \text{Work station 1} \quad \text{WIP1(i)} \quad \text{Work station 2} \quad \text{WIP2(i)} \]

\[ v1(i) \quad v2(i) \quad v3(i) \]

**FIGURE 2.** Production system model consisting of two work stations with work-in-process WIP(i) and product flow velocities v(i).
**Operational rules:** The current work-in-process of every work station is represented by the variable $WIP(i)$ where $i$ is the number of discrete time steps. The product flow into and out of the work station is represented by the product flow velocities $v(i)$, which can be interpreted as work per time unit. Therefore, the variation of the work-in-process of the work stations is indicated by

$$
\Delta WIP(i) = WIP(i+1) - WIP(i) = \tau \cdot (v_{in}(i) - v_{out}(i)) \quad (1)
$$

with $\tau$ as time step size determining the grade of discretisation. One approach to dynamic feedback control is the regulation of the product flow depending on the work-in-process:

$$
v1(i) = COM1 \cdot (WIP_{1\text{max}} - WIP1(i)) \quad (2)
$$
$$
v2(i) = COM2 \cdot (WIP_{2\text{max}} - WIP2(i)) \quad (3)
$$
$$
v3(i) = COM3 \cdot WIP2(i) \quad (4)
$$

The parameters COM are constant, normalized production rates.

**Capacity:** The capacity of manufacturing is described by the production rates of the work stations (COM), which are kept constant. The work-in-process is limited by the maximum buffer level $WIP_{\text{max}}$

$$
0 \leq WIP \leq WIP_{\text{max}} \quad (5)
$$

**Queuing policies:** The global queuing policy is „first in – first out“.

**Order release:** There are no manufacturing orders with due-dates. Only the product flow between two work stations as a part of a more complex production system is considered.

**The Dynamics of the Linear Coupled System**

Under the assumptions named in the previous section, the work-in-process can be described as iterative mechanisms:

$$
WIP1(i+1) = WIP1(i) + \tau \cdot (v1(i) - v2(i)) \quad (6)
$$
$$
WIP2(i+1) = WIP2(i) + \tau \cdot (v2(i) - v3(i)) \quad (7)
$$

The WIP levels change their values according to the sum of preceding WIP levels and inputs and outputs in each iteration step. So the work stations are linearly coupled via the product flow velocity 2. The dynamic behaviour of this system is generated by equations 6 and 7, which can be understood as the operational rules for the manufacturing process.

This system ran in a simulation with the following system parameters and initial conditions:
\[ \tau = 1, \quad \text{WIP}_{1\text{max}} = \text{WIP}_{2\text{max}} = 1, \quad \text{WIP}_{1}(1) = 0.4, \quad \text{WIP}_{2}(1) = 0.9, \quad \text{COM}_{1} = 0.9, \quad \text{COM}_{2} = 0.4, \quad \text{COM}_{3} = 0.4. \]

The system stabilises itself after a few iteration steps (see figure 3 and 4). The state space diagram shows the system state \{WIP_{1}(i), WIP_{2}(i)\} proceeding from the initial value \((0.4, 0.9)\) to the stabilization point \((0.78, 0.5)\). The dynamic behaviour of the system does not depend on the initial values nor on the relation of the production rates COM.

\[ \begin{align*}
\text{FIGURE 3.} & \quad \text{WIP levels of the linear coupled system,} \\
\text{WIP}_{1\text{eq}} &= 0.78, \quad \text{WIP}_{2\text{eq}} = 0.5. \\
\text{FIGURE 4.} & \quad \text{State space of the linear coupled system.}
\end{align*} \]

**The Dynamics of the Non-linear Coupled System**

Non-linear operational rules are introduced to get a number of system states needed for faster adaptation of the system. An example is represented by equations 8 and 9.

\[ \begin{align*}
\text{WIP}_{1}(i+1) &= \text{WIP}_{1}(i) + \tau \cdot (v_{1}(i) - X \cdot \text{WIP}_{1}(i) \cdot v_{2}(i)) \quad (8) \\
\text{WIP}_{2}(i+1) &= \text{WIP}_{2}(i) + \tau \cdot (X \cdot \text{WIP}_{1}(i) \cdot v_{2}(i) - v_{3}(i)) \quad (9)
\end{align*} \]

Here the control of product flow is non-linear by coupling both buffer levels. Additionally, a coupling parameter \(X\) was introduced, which has the dimension of the production rate COM. This coupling parameter allows the controller to choose a tight or loose coupling of the work stations. The dynamic behaviour of this system depends sensitively on the coupling parameter \(X\).

Figure 5 shows the oscillating WIP levels at \(X = 4\). These oscillations are indicated by two fixed points in the state space and a diagonal link between them (see figure 6).
Figure 7 shows two typical bifurcation scenarios, which in this case originate from the coupling of WIP1 and WIP2. Variation of the coupling parameter X shows different qualities of dynamic behaviour. Bifurcation and deterministic chaos occur at large values of X, whereas lower values of X lead to the stabilization of the buffer levels after a small number of iteration steps similar to the linear coupled system (see figure 3 and 4). Depending on the value of the production rate COM2, the coupling parameter X can be used as a control parameter. It permits the controller to operate the system in a stabilized mode or in a multimode. The value of X is limited by the capacity restrictions. Large values deliver results beyond the capacity limits. This results from neglecting the range of the production rates. Interpretation is difficult in this case, but it shows that operation of the system is possible in a wide range of the coupling parameter X.
To satisfy the capacity limits in real production systems, additional restrictions were formulated like “stop on overflow” or “stop if buffer empty”. These restrictions force the system into a stabilized state (see figure 8).

FIGURE 8. WIP levels of the non-linear coupled system with overflow control depending on the coupling parameter X.

Rapid adaptation can be achieved more easily with a non-linear control mechanism. The parameter X can be used to adjust the equilibrium WIP levels in a wide range, allowing the exchange of available capacity between different work stations.

A point of further discussion will be the interpretation of the parameter X regarding capacity control. Changing of X only leads to flow regulation. As in the case of capacity restrictions, limits to the ability of flow changes should be included. This can be done only in a more system specific approach, taking into consideration additional system parameters.

CONCLUSION

A general approach for modelling and control of production systems was introduced that was developed using concepts of Nonlinear Dynamics Theory and PPC research. Manufacturing processes and PPC mechanisms are considered to be a unity that build a dynamical system. The functions of a PPC system have been introduced as functional groups. The possibilities of the dynamical approach have been shown for every group.

A model was discussed in the last section, demonstrating the function of non-linear operational rules, which enable multimode operation (bifurcation, chaos). The application of Nonlinear Dynamics control methods is possible in this case of explicit deterministic chaos.

In most cases, the dynamic behaviour of a production system will not be described only by a set of operational rules. System state definition will be the primary step.
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