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Phase-synchronisation in continuous flow models of production networks

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Abstract

To improve their position at the market, many companies concentrate on their core competences and hence cooperate with suppliers and distributors. Thus, between many independent companies strong linkages develop and production and logistics networks emerge. These networks are characterised by permanently increasing complexity, and are nowadays forced to adapt to dynamically changing markets. This factor complicates an enterprise-spreading production planning and control enormously. Therefore, a continuous flow model for production networks will be derived regarding these special logistic problems. Furthermore, phase-synchronisation effects will be presented and their dependencies to the set of network parameters will be investigated.

Keywords: Production network; Continuous flow model; Synchronisation

1. Introduction

Production networks are distinguished by a permanently growing complexity and are nowadays more than ever forced to adapt fast to dynamically changing markets. These factors complicate an enterprise-spreading production planning and control enormously. Therefore, recent studies of production networks were focussed on the dynamical aspects and it has been discovered that the material transport within those networks can be considered as a physical transport problem (e.g., [1,2]) with balance equations for delivered material, as already mentioned in Ref. [3]. In particular, non-linear behaviour in production systems was investigated [4] and models were found to exhibit complex, oscillatory and even chaotic behaviour [5–7]. Thus, stability analyses of different models and topologies were performed [8,9] to detect critical sets of parameters and stabilising influences. This is in fact very useful for the supression of the bullwhip effect, that describes the amplification of oscillatory amplitudes of delivery rates along the supply chain [10]. But another very interesting and sophisticated field of non-linear dynamics, the phenomenon of synchronisation, has not yet been applied to production systems. It should be investigated, if synchronisation also had such a stabilising effect on the network or helped to avoid the bullwhip effect. It is of further practical economic importance, if

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synchronisation can lead to a better adjustment of production processes within a supply chain or network and, thereby, help to improve the competitiveness of these enterprises.

One of the first reports on synchronisation was made in the 17th century by Huygens [11], who discovered that two pendulum clocks mounted on the same beam oscillated with the same frequency. Since then, synchronisation was found in many disciplines of physics. For instance, Rayleigh found nearby organ tubes to synchronise their frequencies [12] and van der Pol studied the synchronisation of electric circuits [13]. More recently, two semiconductor lasers were found to exhibit synchronisation of their light intensities [14] and for clusters of driven acoustic cavitation bubbles simulations also showed synchronisation effects [15]. These examples and numerical simulations (e.g., Ref. [16]) show that synchronisation is a well-defined and well-examined phenomenon for a large variety of physical applications. But synchronisation occuring in complex systems such as supply chains or supply nets are neither well investigated nor well understood. Hence, in this paper we will derive extensions to a continuous flow model for production networks introduced by Helbing [1], that is based on fluid-dynamics models (e.g., see Ref. [17]). The continuous flow approach has several advantages, but also disadvantages as well, compared to a discrete event approach. Since in production systems mostly discrete products have to be handled, a discrete event simulation (DES) model would be favourable here. But with continuous flow models consisting of a set of differential equations large networks can be modelled with less computational effort than with a DES model. Additionally, continuous flow models can take non-linear interactions into account and are suitable for online control under dynamically changing conditions [2,8].

In contrast to the model introduced by Helbing [1], we will on the one hand restrict the possible network topologies: many companies concentrate on their core competences and start cooperations with suppliers and distributors to improve their position at the market. The consequence for the model is, that only one product can be manufactured in a node. On the other hand, we will focus on existing logistic policies, namely the sealing-off to competitors. Consequently, the only information an enterprise can access to plan and control its production is the demand, respectively, the orders of customers. Based on this model, synchronisation phenomena will be investigated and especially the capability of parameters to change the coupling strength between the nodes will be identified. This coupling is basically bidirectional, but consists of different types in both directions. One is given by the flow of information, e.g., orders or buffer levels, and the other one by the flow of material, e.g., delivered goods or educts. Contrarily, classically coupled systems like Huygens clocks [11] basically only exhibit one kind of coupling, e.g., elastic forces.

2. Production network model

2.1. Basic topology

We will consider a production network consisting of N nodes $i, j \in \{1 \dots N\}$ manufacturing the products $p \in \{1 \dots N\}$. Every node is assumed to produce exactly one product due to its concentration on core competences. So the specification of a node j determines uniquely the product ($p = j$) that is produced there. The nodes are connected by edges which represent the coupling between the nodes. More precisely, the coupling is realised by the flow of material and information along these edges. Whereas, in this model no capacity constraints are implemented, only the delivery time T_{ij} from node i to node j is taken into consideration.

Every node i is composed of an output buffer O_i, where the manufactured goods are stored for delivery, and a production unit, characterised by the production rate Q_i. In order to ensure a continuous production, a safety stock for every incoming product (educt) is established. Thus, node i also contains input buffers I_i^j to store goods from nodes j. Fig. 1 shows the schematic illustration of a network with four nodes and depicts the construction of a single node i. The production process starts with the delivery of the educts from the input buffers to the production unit. Their ratios are given by the coefficient matrix c_i^j. On the other hand, the distribution of products from the output buffers to other nodes is described by the coefficient matrix d_j^i. Naturally, all these coefficients have to fulfil the conditions $0 \leqslant c_i^j \leqslant 1$, $0 \leqslant d_j^i \leqslant 1$ and $\sum_{j=1}^N c_i^j \leqslant 1$, $\sum_{i=1}^N d_j^i \leqslant 1$. To ensure the conservation of the flow, there must be an inflow of resources into the network to every node i, given by $c_i^0 = 1 - \sum_{j=1}^N c_i^j$, and an outflow of products to external consumers, given by $d_j^{N+1} = 1 - \sum_{i=1}^N d_j^i$.

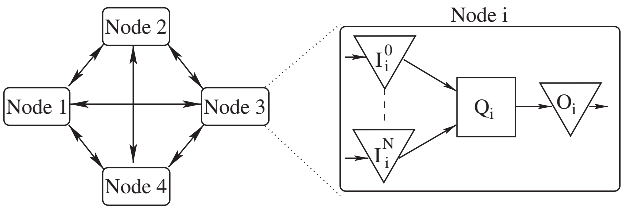

Fig. 1. The left part illustrates schematically the structure of a network. The nodes correspond to companies and the arrows to the flow of material and information within the network. The right part depicts the internal structure of such a node: it consists of input buffers I_i^j, a production unit Q_i and an output buffer O_i.

2.2. Temporal evolution of the network

Following recent works of Helbing [1], balance equations for the temporal evolution of the input and output buffers can be derived:

$$\frac{\mathrm{d}O_i(t)}{\mathrm{d}t} = Q_i(t) - \sum_{j=1}^{N+1} S_{ij}(t), \tag{1}$$

$$\frac{\mathrm{d}I_i^j(t)}{\mathrm{d}t} = S_{ji}(t) - c_i^j Q_i(t), \tag{2}$$

$$S_{ji}(t) = \min\left[\frac{D_{ji}(t)}{T_{ji}}, \frac{d_j^i(t) O_j(t)}{T_{ji}}\right], \tag{3}$$

$$D_{ji}(t) = \max[0, c_i^j Q_i(t) \cdot T_{ji} + c_i^j \delta_{ij}^{buffer}(t)]. \tag{4}$$

Thereby, $S_{ji}(t)$ is the supply of products from node j to node i and $D_{ji}(t)$ the demand. When calculating the demand, additionally to $c_i^j Q_i(t) \cdot T_{ji}$, the difference $\delta_{ij}^{buffer}(t)$ of the actual buffer level to a desired one is taken into consideration to provide enough educts for production in case of a shortage. To satisfy the demand, the production rate $Q_i(t)$ is adapted to a desired one, determined with the function $Q_i^{opt}(t)$. The adaptation is then achieved by an exponential, but to $Q_i^{opt}(t)$ limited growth

$$\frac{\mathrm{d}Q_i(t)}{\mathrm{d}t} = \tau_i \cdot Q_i(t) \cdot \frac{Q_i^{opt}(t) - Q_i(t)}{Q_i^{opt}(t)}. \tag{5}$$

This differential equation leads to the so-called logistic function. In its inflexion point at $t = 0$ the slope is maximum and so the production rate will grow maximally and then converge to optimal value Q_i^{opt}. τ_i represents a characteristic constant for every node, that determines the speed of growth. Fig. 2 shows the normalised logistic function for different parameters τ. Due to capacity constraints and economic reasons, the production rate must remain in the interval of a maximum and a minimum production rate $Q_i^{min} \leqslant Q_i(t) \leqslant Q_i^{max}$. This will be taken into account when calculating Q_i^{opt} (see Eq. (9)). Also, non-negativity of the input buffers must be granted by limiting $Q_i(t)$, respectively, $\mathrm{d}Q_i(t)/\mathrm{d}t$:

$$\frac{\mathrm{d}Q_i(t)}{\mathrm{d}t} \leqslant -Q_i(t) + \min_{j, c_i^j \neq 0}\left[\frac{1}{c_i^j}\left(I_i^j(t) + \frac{\mathrm{d}I_i^j(t)}{\mathrm{d}t}\right)\right]. \tag{6}$$

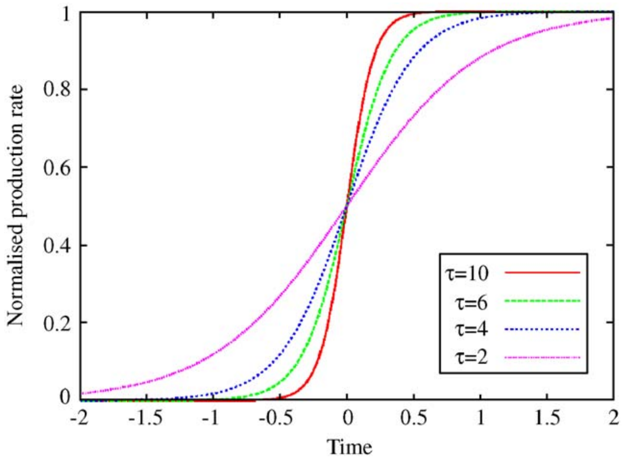

Fig. 2. Visualisation of the time-dependent adaptation of the production rate (Eq. (5)) is shown for different values of the adaptation constant τ. The integration of that differential equation leads to the logistic function.

The last time-dependent value is the delivery coefficient. It is linearly adapted to a desired value $g_{ji}(t)$, which is calculated with certain logistic policies (see Eq. (11)). The adaptation time ϑ_j is characteristic for every node

$$\frac{\mathrm{d}d_j^i(t)}{\mathrm{d}t} = \frac{1}{\vartheta_j}[g_{ji}(D_{ji}(t)) - d_j^i(t)]. \tag{7}$$

2.3. Logistic policies

The logistic policies determine the desired production rate, delivery coefficients, input buffer level and thereby the functions $\delta_{ij}^{buffer}(t)$, $Q_i^{opt}(t)$ and $g_{ji}(t)$. In order to keep the production running always, a certain safety stock in the input buffers is needed. Therefore, the averaged overall demand from other nodes and additionally a safety term has to be in the input buffers to ensure a continuous production in future. This includes the time-averaged demand D_{ij}^{avg} multiplied with the replenishment time and an additional amount to ensure the delivery reliability, which defines the safety factor z_i (see Ref. [18]). Thereby, D_{ij}^{std} is the standard deviation of the demand.

$$\delta_{ij}^{buffer}(t) = \sum_{k=1}^{N+1} (T_{ik} D_{ik}^{avg} + z_i \cdot D_{ik}^{std} \sqrt{T_{ik}}) - I_i^j(t). \tag{8}$$

For average and standard deviation, a gliding window of fixed size is used. Thus, different ordering strategies are possible. A fast reaction to changes corresponds to a small window size and a slow adaptation to larger windows.

The optimal production rate depends on the overall demand for produced goods ($\sum_{j=1}^{N} D_{ij}/T_{ij}$) and on an optimal output buffer level O_i^{opt}:

$$Q_i^{opt}(t) = \sum_{j=1}^{N+1} \frac{D_{ij}(t)}{T_{ij}} + \left(1 - \left(\frac{O_i(t)}{O_i^{opt}(t)}\right)^{k_i}\right) \cdot Q_i^{max}. \tag{9}$$

As already mentioned, the production rate must remain in the interval $Q_i^{min} \leqslant Q_i(t) \leqslant Q_i^{max}$. This can be ensured by forcing $Q_i^{opt}(t)$ to be in the same interval, because $Q_i^{opt}(t)$ will be reached asymptotically (see Eq. (5)). To achieve the optimal value for the output buffer, the relative buffer level $O_i(t)/O_i^{opt}(t)$ is exponentiated by a constant $k \in [1, \infty[$. This reflects a certain filling strategy: a value of $k = 1$ means that the desired production rate equals the amount needed to reach $O_i^{opt}(t)$. The case $k > 1$ represents an overproduction to reach the optimal buffer level faster and to have a larger safety stock.

The value for the optimal output buffer level is calculated analogously to the optimal input buffer level in Eq. (8): the overall average demand and the overall standard deviation of the demand are taken into account. Again, z_i denotes a certain delivery reliability

$$O_i^{opt} = \sum_{j=1}^{N+1} T_{ij} D_{ij}^{avg} + z_i \cdot \sum_{j=1}^{N+1} D_{ij}^{std} \sqrt{T_{ij}}. \tag{10}$$

To adapt the delivery coefficients, the ratio of the gliding average of one node's demand to the total demand is considered

$$g_{ji}(t) = \frac{\int_0^{\theta_i} D_{ji}(t-\tilde{t})\,\mathrm{d}\tilde{t}}{\sum_{i=1}^{N+1} \int_0^{\theta_i} D_{ji}(t-\tilde{t})\,\mathrm{d}\tilde{t}}. \tag{11}$$

The time horizon θ_i has the same properties like the window size of the gliding average D_{ji}^{avg} and the gliding standard deviation D_{ji}^{std} in Eqs. (8) and (10): a fast reaction to changes corresponds to a small window size and a slow adaptation to larger windows. Furthermore, it is assumed that a node uses the same policies (i.e., time horizons) for all of the three quantities.

3. Simulation results

The developed model was analysed for synchronisation effects with the focus on the coupling ability of the different parameters, which are on the one hand the parameters fixed by the topology: namely the delivery time T_{ij} and the coefficient matrix c_i^j. Additionally, every node in the network is assumed to prefer the same delivery reliability of 95%, so that $z = z_i = 1.64$. And on the other hand, there are the parameters determined by the logistic policies: τ_i, ϑ_i, k_i and θ.

Their effect on the oscillations and, in particular, on the synchronisation of the nodes was investigated in a linear supply chain with three nodes. Thereby, node 1 consumed external resources for its production. Node 2 was delivered by node 1 and supplied node 3 with products. Node 3, in turn, had to satisfy an external demand. Since the external demand in general varies, e.g., seasonal fluctuations, it was realised by the simplest regularly varying function, a sine function, here with a period length $T = 15.7$ of arbitrary time units and amplitude of 1 around a mean value of 3.

The adaptation of the delivery coefficient d_j^i was not necessary in a linear supply chain, so a variation of ϑ_i would not change the oscillatory behaviour. Simulation results also showed that in this example a change of the time horizon θ did not influence the oscillation either. Thus, the parameter τ_i which determines the adaptation speed of the production rate $Q_i(t)$ was changed, while all the other parameters remained constant. Fig. 3 shows the oscillation of the single nodes' input buffers for $\tau_i = 0.2$ and 1. Thereby, $\tau_i = \tau$ was constant for the different nodes in one scenario. It is clearly visible in every subfigure that the input buffer level amplitudes increase along the supply chain, which is known as the "bullwhip effect" [10]. To identify a

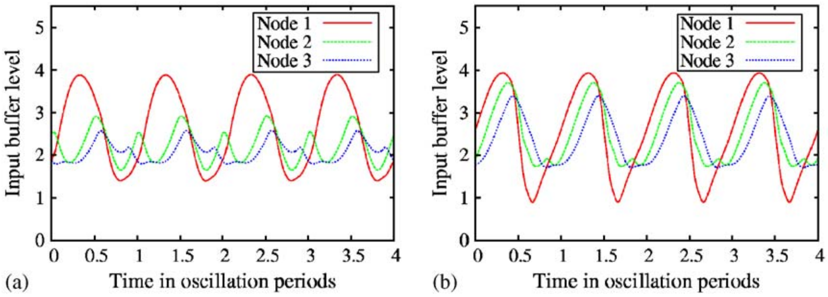

Fig. 3. Oscillation of the nodes' input buffers simulated with Eqs. (1)–(11) and the parameters $T_{ij} = \mathbf{1}$, $\vartheta_i = 3$, $z_i = 1.64$, $k_i = 1$, $\theta_i = 30$ and different values for τ_i are shown. The bullwhip effect [10], i.e., the amplification of buffer levels along the supply chain, is visible. (a) $\tau = 0.2$, (b) $\tau = 1$.

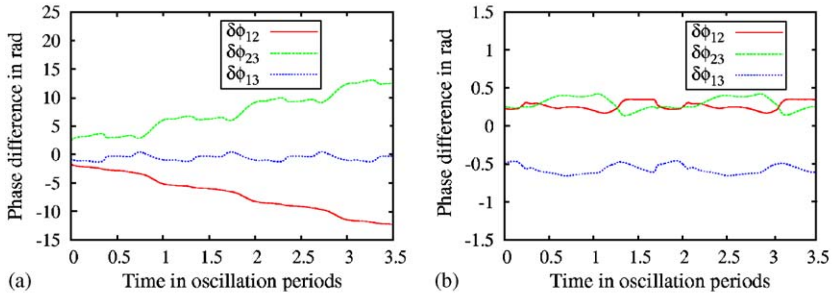

Fig. 4. The phase differences between the single nodes are shown. Phases were determined via the analytic signal approach and computed with a hilbert transform (cf. Eqs. (12) and (13)). In part (a), with $\tau = 0.2$, only phase difference $\delta\Phi_{12}$ is bounded and consequently, only nodes 1 and 2 have synchronised their phases. Additionally, the graph of $\delta\Phi_{23}$ shows phase slips of π after every oscillation period. Only an imperfect phase synchronisation is visible. In part (b), with $\tau = 1$, all phase differences are bounded and so a perfect phase synchronisation was established. (a) $\tau = 0.2$, (b) $\tau = 1$.

possible phase synchronisation the analytic signal [19] of the oscillations was computed via a hilbert transform [20,21]

$$\xi(t) = s(t) + \mathrm{i} \cdot s_h(t) = A(t) \cdot \mathrm{e}^{\mathrm{i}\Phi(t)}, \tag{12}$$

$$s_h(t) = \pi^{-1} P.V. \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} \, \mathrm{d}\tau \tag{13}$$

and then the phase $\Phi(t)$ was extracted. As is known, a phase synchronisation is existent if the phases of both oscillations are locked, i.e., $\delta\Phi_{12} = |n\Phi_1 - m\Phi_2| \leqslant const$ (e.g., see Ref. [16]). Here, the indices denote the index of the nodes, i.e., $\delta\Phi_{12}$ is the difference between nodes 1 and 2.

All possible phase differences between the three oscillations are shown for $\tau = 0.2$ in Fig. 4(a) and for $\tau = 1$ in Fig. 4(b). It can clearly be seen that for $\tau = 1$ a phase synchronisation in the hole supply chain was established, because all phase differences are bounded to $|\delta\Phi_{12}| < 0.5$, $|\delta\Phi_{13}| < 0.7$ and $|\delta\Phi_{23}| < 0.5$ (see Fig. 4(b)). In contrast, for $\tau = 0.2$ only one phase difference is bounded ($|\delta\Phi_{13}| < 1.5$) and the other two ($|\delta\Phi_{12}|$, $|\delta\Phi_{23}|$) are not (see Fig. 4)(a). For $\delta\Phi_{23}$, after every oscillation period a phase slip can be observed, so that only an imperfect phase synchronisation was established. Further simulations also showed that a variation of parameter k, while the other parameter remained constant, did not have a similar effect on synchronisation, even not on the oscillatory behaviour. A change from $k = 1$ to 20 did only marginally effect the oscillation and thus, the synchronisation behaviour did not change at all in this configuration.

4. Conclusion

In our derived continuous flow model for production networks, applied to a linear supply chain with three nodes, phase synchronisation was found. But the occurrence of synchronisation strongly depended on the parameter τ, which determines the adaptation speed of the production rate. For values $\tau = 1$ and larger, perfect phase synchronisation of all nodes could be observed. Contrarily, values of $\tau = 0.2$ and smaller did not lead to a synchronisation of all nodes. In the given example, one phase difference indicated an established phase synchronisation, whereas the other two differences only showed an imperfect phase synchronisation with phase slips and no visible synchronisation. In contrast, parameter k did not affect the synchronisation at all. These first simulation results show that a stronger adaptivity to varying demands seems to increase the synchonisability of the nodes. But still large variations in the buffer levels and also the bullwhip effect are present. That is due to economic reasons not desirable. Thus in future work, mechanisms to reduce these effects by synchronisation have to be found. The results presented here lead to the assumption that in more complex topologies synchonisation of single or even of all nodes can be found.

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